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A note on the Nusselt number adjacent to a vertical isothermal plate immersed in thermally stratified water at low temperatures

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Abstract

The local Nusselt number of a buoyancy induced boundary layer adjacent to an isothermal vertical plate immersed in a linearly thermally stratified water at low temperatures is investigated. The results are obtained with direct numerical solution of the boundary layer equations. Both upward and downward flow is considered. When the plate temperature is equal to maximum density temperature the Nusselt number shows a maximum. It was also observed that the Nusselt number curve changes from linear to nonlinear form as the plate temperature varies from the linear to nonlinear temperature range.

Keywords: Isothermal vertical plate; Free convection; Stratified water; Density extremum

1. Introduction

Natural convection along a vertical isothermal plate immersed in an infinite and quiescent ambient fluid with uniform temperature forms a classical problem that has been solved 50 years ago. In many instances, however, the ambient fluid is not isothermal. Such flows can be found in nature and technology. For example the atmosphere and the ocean are thermally stratified. In energy storage systems such as ponds or in heat transfer from bodies in enclosures the thermal input itself produces the stratification. There are works in the literature which deal with the problem where both the plate and the ambient fluid are nonisothermal. In the present work we concentrate precisely on natural convection along a vertical isothermal plate immersed in a thermally stratified ambient fluid.

Eichhorn (1969) studied the effect of linear thermal stratification on the heat transfer of a vertical plate and obtained solutions with the series expansions method. Chen and Eichhorn (1976) restudied the above problem and after concluding that a similarity solution was not possible, they treated the problem using the local nonsimilarity method. Raithby and Hollands (1978) applied an approximate method and showed good agreement in predicting the Nusselt number with experimental data. Venkatachala and Nath (1981) solved the complete set of governing differential equations using a finite difference method. The existence of similarity solution for this problem was considered impossible for a long time until Kulkarni et al. (1987) succeeded to derive a solution of this kind. Henkes and Hoogendoorn (1989) presented also a similarity solution. Krizhevsky et al. (1996) studied the stability characteristics of a buoyancy induced flow along an isothermal vertical plate immersed in a linear ambient thermal stratification. Tanny and Cohen (1998) presented the most complete experimental results for this problem. The results concern the temperature field in a vertical isothermal plate immersed in linearly stratified water at temperatures between 25 and 38 °C.

All the above mentioned works concern fluids with linear relationship between density and temperature like air and water at high temperatures. However, the water density–temperature relationship is nonlinear at low temperatures. The density of pure water is maximum at 3.98 °C. The density increases as the temperature decreases approaching 3.98 °C, while the density decreases as the temperature decreases from 3.98 to 0 °C (see Fig. 1). The purpose of the present note is to present the Nusselt number variation along a vertical isothermal

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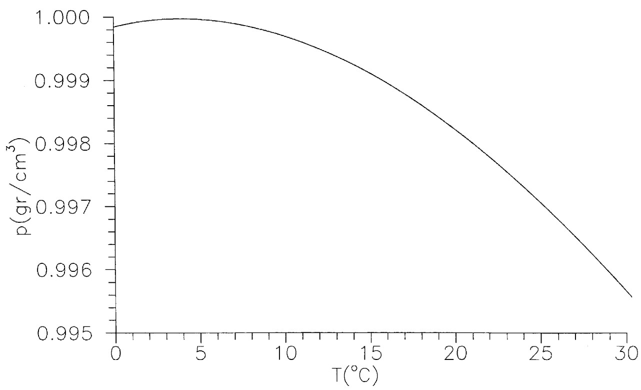

Fig. 1. Variation of water density in the 30–0 °C region.

plate immersed in a linearly stratified water at low temperatures. At this range the linear thermal stratification corresponds to nonlinear density stratification and for that reason the Nusselt number departs from the classical variation which corresponds to linear density–temperature relationship.

The Nusselt number has been obtained with the direct solution of the boundary layer equations using the method described by Patankar (1980). All water thermophysical properties (dynamic viscosity μ, thermal conductivity k, specific heat c_p and density ρ) have been considered as functions of temperature during the solution procedure. The international equation of state for seawater (Fofonoff, 1985) has been used for the calculation of density while μ, k and c_p have been calculated using the formulae given by Kukulka et al. (1987). The finite difference method is used with primitive coordinates x, y and a space marching procedure is used in x direction with an expanding grid. More information about the equations and the solution procedure may be found in Pantokratoras (2001, 2002). The only difference of the present work compared to the last two works are the boundary conditions which were as follows:

at $y = 0 \quad u = v = 0,\ T = T_w$

as $y \to \infty \quad u = 0,\ T = T_\alpha$

where the ambient temperature varies according to the following equation:

$$T_\alpha = T_\alpha(0) + x\mathrm{d}T_\alpha/\mathrm{d}x \tag{1}$$

In the above relations T_w is the plate temperature, T_a is the local ambient temperature, $\mathrm{d}T_a/\mathrm{d}x$ is the stratification parameter and T_a (0) is the ambient temperature at the plate edge.

2. Results and discussion

The local Nusselt number is defined as

$$Nu_x = -\frac{x}{T_w - T_a}\left[\frac{\partial T}{\partial y}\right]_{y=0} \tag{2}$$

The derivative $[\partial T/\partial y]_{y=0}$ was calculated from the following equation:

$$\left[\frac{\partial T}{\partial y}\right]_{y=0} = \frac{T(2) - T(1)}{\Delta y} \tag{3}$$

where (1) is a grid point on the plate and (2) is an adjacent grid point along y.

The accuracy of the method was tested comparing the results with the most recent experimental data given by Tanny and Cohen (1998). In Fig. 2 the downstream variation of the local Nusselt number is given for a vertical isothermal plate with $T_w = 32.6$ °C, $T_a(0) = 25.95$ °C and $\mathrm{d}T_a/\mathrm{d}x = 0.42$ °C/cm. These data correspond to an experiment conducted by Tanny and Cohen (Fig. 5 in their work). At the same figure the theoretical prediction by Kulkarni et al. (1987) is included. From this figure it is seen that the present method compares well with the experimental data and gives a little better results than that of Kulkarni et al. We tried three different values of Δx (0.001, 0.005 and 0.010 cm) in order to obtain grid independent results. The values of the Nusselt number corresponding to $\Delta x = 0.001$ cm and $\Delta x = 0.010$ cm differed about 0.5 %. All the results of the present work have been produced with $\Delta x = 0.001$ cm and are grid independent.

In Fig. 2 there is another curve (dashed line) which shows the variation of temperature gradient $[\partial T/\partial y]_{y=0}$ along the plate. There is an anomaly of the temperature gradient near the plate leading edge but as x increases the temperature gradient acquires normal values. It should be noted here that the anomaly near the plate leading edge appears in every boundary layer simulation with a parabolic numerical procedure like that of Patankar (1980). The solution procedure starts with a boundary layer with zero thickness and the boundary layer takes its complete form at some downstream

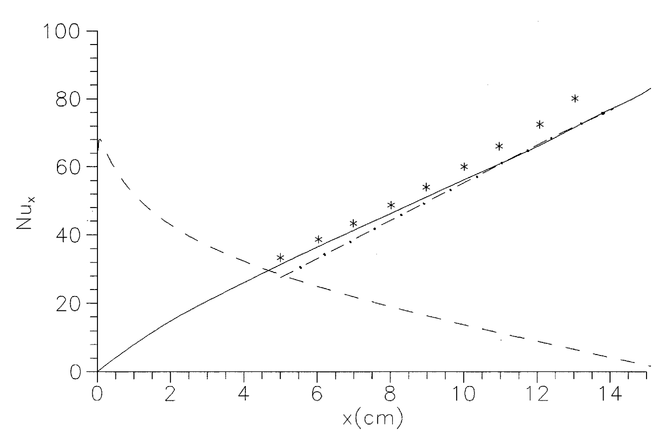

Fig. 2. Downstream variation of the local Nusselt number: *, experimental data by Tanny and Cohen for $T_w = 32.6$ °C, $T_a(0) = 25.95$ °C and $\mathrm{d}T_a/\mathrm{d}x = 0.42$ °C/cm; dashed line with dots, theoretical prediction by Kulkarni et al.; solid line, theoretical prediction by the present method; dashed line, theoretical prediction of temperature gradient $(\partial T/\partial y)_{y=0}$.

position from the leading edge. This problem appears also in the simulation of other boundary layer flows like jets and plumes. From Fig. 2 it is seen that, at high water temperatures where the density–temperature relationship is linear, the temperature gradient decreases almost linearly whereas the local Nusselt number increases almost linearly with x.

In the present work we deal only with stable stratification. We consider a water layer with temperature at the bottom equal to 4 °C and assume that the water temperature increases linearly upwards. This kind of linear temperature stratification corresponds to a density stratification that is nonlinear near the bottom and changes to linear as the temperature increases upwards. If we consider the problem of an isothermal plate with temperature greater than 4 °C (say for example 10 °C) and ambient temperature at the plate edge equal to $T_a(0) = 4$ °C the water will move upwards in a stratified ambient which changes from nonlinear to linear. In this case the Nusselt number curve is almost linear as happens in Fig. 2. More interesting is the opposite situation where the water moves in a stratified environment which changes from linear to nonlinear. We consider again the previously mentioned stably stratified water layer, a plate temperature equal to 4 or 5 °C and ambient temperature at the plate edge greater than the plate temperature (for example $T_a(0) = 11$ °C). We assume that the plate edge lies higher than the layer bottom and the distance x is positive from the plate edge downwards. In this case the water near the plate is cooler than the ambient water and the fluid moves downwards. For the simulation of this situation the stratification rate dT_a/dx in Eq. (1) is taken negative because now the ambient temperature decreases as x increases. Now we have a case where the fluid moves in a stratified environment which changes from linear to nonlinear. The results are shown in Figs. 3 and 4.

In Fig. 3 the downstream variation of the local Nusselt number and of the local temperature gradient is given for a vertical isothermal plate with $T_w = 4$ °C, $T_a(0) = 7$ °C and different dT_a/dx. Laying the temperature gradient at the same figure which is intended for the Nusselt number has caused a compression and distortion of the temperature gradient curves. However, a careful examination of these curves reveals that the temperature gradient varies completely nonlinear along the plate and this nonlinearity causes a nonlinear variation of the Nusselt number. The Nusselt number increases, reaches a maximum and then decreases. We also see that as the absolute stratification rate increases the Nusselt number decreases.

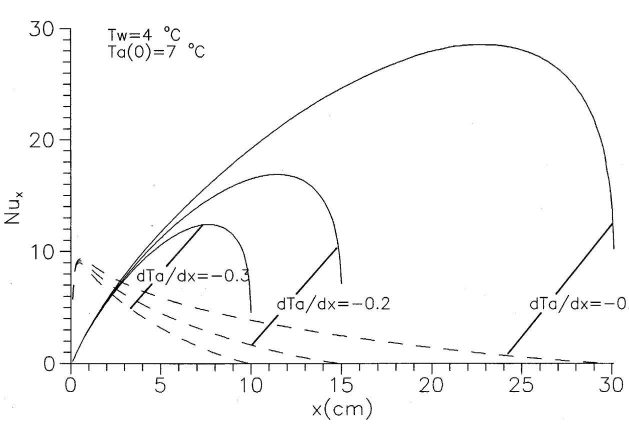

Fig. 3. Theoretical prediction of the local Nusselt number for $T_w = 4$ °C, $T_a(0) = 7$ °C and different stratification rates dT_a/dx (°C/cm); dashed lines correspond to temperature gradient $(\partial T/\partial y)_{y=0}$.

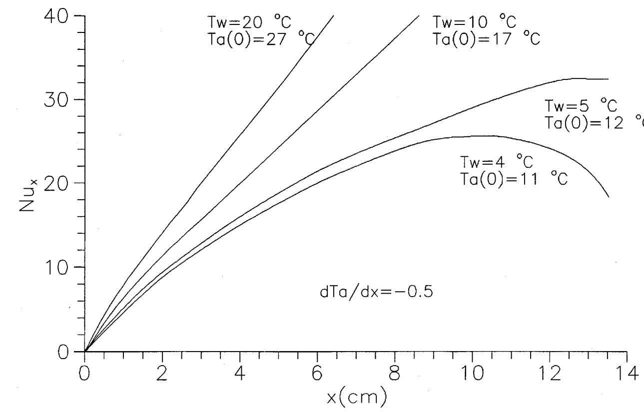

Fig. 4. Theoretical predictions of the local Nusselt number for $dT_a/dx = -0.5$ °C/cm and different plate temperatures. The initial temperature difference is 7 °C for all cases.

In Fig. 4 the downstream variation of the local Nusselt number is given for a fixed stratification rate, a fixed initial temperature difference ($\Delta T = 7$ °C) and different plate temperatures (4, 5, 10 and 20 °C). It is seen that as the plate temperature varies from the nonlinear to linear region the Nusselt number curve tends from nonlinear to linear form.

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